ζ_r damping matrix (11×11):

$$\zeta_r = \text{diag}\,\{0\ \ 0\ \ 0.005\ \ldots\ \ 0.005\} \tag{M10}$$

Scalars in Eq. (6):

$$\tilde{m}_r = 1.6,\ \tilde{I}_{rx} = 0.0021328,\ \tilde{I}_{ry} = 0.013333,\ \beta = 1,$$

$$\tilde{\beta} = \dot{\tau}^2, \dot{\tau} = 5.5885\text{E}-04 \tag{M11}$$

$\tilde{R}$ weighting matrix (10×10):

$$\tilde{\boldsymbol{R}} = 15.6097\ \boldsymbol{R} \qquad \boldsymbol{R} \text{ is at the user's discretion} \tag{M12}$$

Having known all the matrices and the scalars constituting the generic model (13) for the control objective (6) an (sub)optimal controller can be synthesized.

Regarding the quality of the model, we mention that from the standpoint of linear momentum about the z axis and angular momentum about the x and y axes the model is (0.4518, 0.0202, 0.8250) complete, and from the viewpoint of Modal Cost Analysis it is 0.8626 complete; the ideal value of these indexes is unity. The reader is urged to consult Hablani[3] and Skelton et al.[2] to appreciate the significance of the above indexes.

Concluding Remarks

This paper offers a generic model to help evolve a composition of techniques particularly suitable for advanced elastic spacecraft to design a suboptimal controller by using the multivariable control theory. The model deals with both the attitude control of a rigid body and the shape control of an elastic structure.

Acknowledgments

We gratefully acknowledge the numerous encouraging discussions with Prof. P.C. Hughes, Institute for Aerospace Studies, University of Toronto, and with Prof. R.E. Skelton, School of Aeronautics and Astronautics, Purdue University. This research is funded by Jet Propulsion Laboratory under Contract No. 955639.

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AIAA 78-1285R

Attitude Stabilization of Large Flexible Spacecraft

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I. Introduction

CURRENTLY, there is undeniable interest in very large spacecraft and satellites; the proposed NASA Satellite Solar Power Station is one of many such concepts.[1,2] These spacecraft are mechanically very flexible and, hence, require a large number of vibration modes to describe their behavior. Among the number of theories developed for control of these highly flexible structures is one developed by Balas.[3,4]

In this paper, we apply this theory to a reasonable model of a large spacecraft and develop a controller design to attitude-stabilize the spacecraft, suppress vibrations, and maintain a high level of pointing accuracy. We illustrate the problems of digitally implementing controllers for flexible spacecraft and point out solutions that make the designs implementable.

This paper includes numerical results obtained by Ginter in his M.S. Thesis.[5]

II. Model for Flexible Spacecraft

The generic problem of actively controlling very large nonspinning spacecraft can best be illustrated by considering the structure to be very longitudinally flexible while having much less lateral flexibility (i.e., confining the flexibility to a single plane). This is a reasonable model for many satellites, and it contains the basic elements of all flexible structures. The most flexible such structure is the Euler-Bernoulli beam with free-end conditions. With appropriate boundary conditions, these dynamics are given by the following partial-differential equation:

$$mu_{tt} + EIu_{xxxx} = F \tag{1}$$

where $u(x,t)$ is the transverse displacement, $F(x,t)$ is the force distribution on the beam, and the beam parameters are the mass density m, the moment of inertia I, the coefficient of elasticity E, and the length L.

The mode shapes $\phi_k(x)$ for this beam model can be obtained as linear combinations of regular and hyperbolic trigonometric functions, and the mode frequencies λ_k can be determined from a transcendental equation.[5] The motion of the beam $u(x,t)$ can be expanded as

$$u(x,t) = \sum_{k=1}^{R} u_k(t)\,\phi_k(x) \tag{2}$$

Presented as Paper 78-1285 at the AIAA Guidance and Control Conference, Palo Alto, Calif., Aug.6-8, 1978; submitted June 23, 1980; revision received Feb. 11, 1981.

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where R is very large (infinite, in theory). The mode amplitudes $u_k(t)$ satisfy

$$\ddot{u}_k + \lambda_k u_k = F_k \tag{3}$$

where

$$F_k \equiv \int_\Omega F\phi_k \mathrm{d}x$$

By forming the state $v = [u^T, \dot{u}^T]^T$, where $u = [u_1, \ldots, u_R]^T$, the spacecraft model becomes

$$\dot{v} = Av + F \tag{4}$$

with

$$A \equiv \begin{bmatrix} 0 & I_R \\ \Lambda_R & 0 \end{bmatrix} \qquad \Lambda_R \equiv \mathrm{diag}[\lambda_1 \lambda_2, \ldots, \lambda_R]$$

and

$$F \equiv [F_1, \ldots, F_R]^T$$

III. Control of Large Spacecraft

Control is introduced to the model by M-point actuators:

$$F(x,t) = \sum_{i=1}^{M} \delta(x - x_i) f_i(t) \tag{5}$$

which produce

$$F = Bf \tag{6}$$

where $f = [f_1, f_2, \ldots, f_M]^T$, the vector of actuator amplitudes, and

$$B \equiv \begin{bmatrix} 0 \\ B_R \end{bmatrix} \qquad B_R \equiv \begin{bmatrix} \phi_1(x_1)\ldots\phi_1(x_M) \\ \vdots \\ \phi_R(x_1)\ldots\phi_R(x_M) \end{bmatrix}$$

Observation is done with P-point sensors of the following types: 1) translational position $[u(z_j,t)]$, 2) rotational position $[\theta(z_j,t) = u_x(z_j,t)]$, 3) translational velocity $[u_t(z_j,t)]$, 4) rotational velocity $[\theta_t(z_j,t) = u_{xt}(z_j,t)]$.

Combinations of these sensors lead to

$$y = Cv \tag{7}$$

where $y = [y_1, \ldots, y_p]^T$, the vector of sensor outputs, and C contains mode shapes $\phi_k(x)$ and their spatial derivatives $\phi'_k(x)$ evaluated at the sensor location z_j.

The fundamental control problem for the system, Eqs. (4), (6), and (7), is that the dimension R of the model is usually very large, but on-board computer capability is limited; hence, the controller must have a substantially smaller dimension N. This means that only N modes can be controlled where $N < R$, and we shall design a controller for the subsystem (A_N, B_N, C_N) rather than the full system (A,B,C). This leads to a major difficulty in flexible system control, which we call spillover. The controller pumps energy into the residual (or uncontrolled) modes and the sensors detect this excitation; these phenomena are called "control spillover" and "observation spillover," respectively, and their combined effect can produce instabilities in the actively controlled spacecraft unless the effect is counteracted.[4]

In the next section, we design the active controller, and, in the last section, we discuss the effects of spillover and digital implementation of the controller.

IV. Active Controller Design

The active controller for the subsystem (A_N, B_N, C_N) consists of a state estimator and a liner feedback control law. When (A_N, C_N) is observable, the state estimator has the form

$$\dot{\hat{v}}_N = (A_N + B_N G_N - K_N C_N)\hat{v}_N + K_N y \qquad \hat{v}_N(0) = 0 \tag{8}$$

where the estimator gains K_N are chosen so that the estimator error $e_N \equiv \hat{v}_N - v_N$ decays at a desired rate. In order to keep observation spillover from entering the state estimator, a prefilter may be used on the sensor outputs; this prefilter could be a comb filter tuned to the controlled-mode frequencies (e.g., using phase-locked loops). The prefilter would reduce the observation spillover and, thus, allow the estimator to track the controlled-mode states. Another approach, developed by Skelton and Likins,[6] augments the state estimator with a very low-dimensional model which approximates the residual mode effects and attempts to subtract them before they can contaminate the control commands.

The control law has the form

$$f = G_N \hat{v}_N \tag{9}$$

where the control gains G_N are determined as though the actual state v_N were present instead of the estimate $\hat{v}_N$. Although these gains can be determined by pole allocation methods, in this application we believe that the steady-state regulator approach makes much more physical sense. We begin with the quadratic cost function

$$J = \int_0^\infty (v_N^T Q_N v_N + f^T R f)\,\mathrm{d}t \tag{10}$$

where R is the positive definite control-weighting matrix and the state-weighting matrix Q_N may be obtained in several ways. If we wish to suppress vibrations in all the controlled modes, we take

$$Q_N = \begin{bmatrix} \Lambda_N & 0 \\ 0 & I_N \end{bmatrix}$$

so that $v_N^T Q_N v_N = E_N$, the energy in the controlled modes. A variation on this approach can be used to emphasize the energy in certain more critical modes. If we wish to have high pointing accuracy, then we must minimize the pointing error $\theta(½,t)^2 + \dot{\theta}(½,t)^2$. This requires

$$Q_N \equiv \begin{bmatrix} c_N c_N^T & 0 \\ 0 & c_N c_N^T \end{bmatrix}$$

where $c_N^T = [\phi'_1(½), \ldots, \phi'_N(½)]$. Of course, variations are possible on this approach if pointing accuracy is required at more than a single location on the spacecraft.

Both of these basic approaches permit calculation of control gains from steady-state regulators (using Potter's method[7] to solve the corresponding Riccati equation) with physically reasonable cost functions, Eq. (10). A sufficient condition for the calculation of these gains is that (A_N, B_N) be controllable and that (A_N, H_N) be observable where $Q_N = H_N^T H_N$.

The controllability and observability conditions for (A_N, B_N, C_N) are affected by the types of sensors and actuators and their placement on the spacecraft.

We have the following:

1) A pair of force actuators can produce a controllable system for the model, Eqs. (4) and (6).

2) For observability, a pair of position sensors will suffice for the model, Eqs. (4) and (7).

However, when velocity sensors are used, there must be position sensor observability and no zero frequencies in the controlled modes. This means that velocity sensors alone cannot observe the rigid-body modes of the model, Eqs. (4) and (7). These results follow from Theorems 2.2-2.5 of Ref. 4.

V. Implementation of Active Controllers

In this section, a particular model of the system, Eq. (1), is chosen for analysis. The beam has length $L=10$ m, mass density $m=15$ kg/m, and a structural rigidity $EI=11{,}820$ kg(m)^3/s^2. This particular choice of parameters results in the first vibrational mode having a natural frequency near 1 Hz. Control of the system is achieved with two bidirectional reaction jets located at $x_1=2$ m and $x_2=8$ m; the nonlinear operation of the jets is not taken into consideration here. Observation of the system is made with three sensors: two rate integrating gyros at $z_1=3$m and $z_2=7$ m to measure rotational deflection at these points, and one displacement sensor at the midpoint of the beam to measure translational deflection. No sensor or actuator dynamics are considered here, but they could easily be added to the model at a later time.

We assume that the primary objective of the control system is to maintain the beam deflection $u(x,t)$ near a nominal value of zero for all points along the beam and to use acceptable levels of control action in this process. This is a variation on the pointing accuracy method of Sec. IV, and can be achieved by selecting a finite number of points spaced at 1 m separation along the beam and producing a vector performance measure $r(t)$ whose entries are

$$r_j(t)=u(x=j-1,t) \qquad (j=1,2,...,11)$$

We choose to control the first five modes ($n=5$) and obtain a cost function of the form, Eq. (10):

$$J=\int_0^\infty (r_N^T r_N+\epsilon f^T f)\,dt$$

where the factor ϵ is chosen to be 4×10^{-6} in order to keep the control forces under 500 N, and the performance vector is r_N, the truncated version of r:

$$r_N=D_N v_N$$

where

$$D_N=\begin{bmatrix} \phi_1(0)...\phi_5(0) \\ \vdots \\ \phi_1(10)...\phi_5(10) \end{bmatrix}$$

and in Eq. (10), we have $Q_N=D_N^T D_N$. From Sec. IV, we see that all controllability and observability criteria are satisfied, and we calculate the control gains by Potter's method:

$$G_N=[G_N^1 G_N^2]$$

where

$$G_N^1=\begin{bmatrix} -370.78 & 406.19 & 342.27 & 166.98 & 84.69 \\ -370.78 & -406.19 & -342.27 & -166.98 & 84.69 \end{bmatrix}$$

and

$$G_N^2=\begin{bmatrix} -136.96 & 138.10 & -40.79 & 23.13 & 12.71 \\ -136.96 & -138.10 & -40.79 & -23.13 & 12.71 \end{bmatrix}$$

We make the reasonable assumption that the sensor noise and random system disturbances are small enough to allow the use of the deterministic state estimator, and we calculate the estimator gains so that the estimator is slightly faster than the system it is controlling. The gains used here are $K_N=[(K_N^1)^T(K_N^2)^T]^T$ where

$$K_N^1=\begin{bmatrix} 4.87 & -51.74 & -4.87 \\ -13.66 & 0.0 & -13.66 \\ 6.04 & 0.46 & -6.04 \\ 11.68 & 0.0 & 11.68 \\ -1.79 & -0.17 & 1.79 \end{bmatrix}$$

$$K_N^2=\begin{bmatrix} 21.99 & -215.45 & -21.99 \\ -63.20 & 0.0 & -63.20 \\ 31.88 & 5.96 & -31.88 \\ 722.8 & 0.0 & 722.8 \\ -57.63 & -7.95 & 57.63 \end{bmatrix}$$

When no spillover is present, the controlled mode poles are $-2.78\pm j2.81$, $-3.00\pm j2.97$, $-0.25\pm j6.27$, $-0.43\pm j17.32$, $-0.35\pm j33.94$, and the estimator poles are $-8.17\pm j1.17$, $-9.77\pm j4.28$, $-11.41\pm j6.56$, $-8.03\pm j17.33$, $-4.27\pm j34.07$.

We introduce spillover by adding the next three modes to the system without redesigning the active controller. If spillover is small, the above poles would remain near their locations and the residual mode poles near $0.0\pm j56.10$, $0.0\pm j83.81$, $0.0\pm j117.05$.

However, if spillover is not small, all system poles are shifted and they become -6.82, -7.55, -4.00, -14.05, $-2.76\pm j.84$, $-2.79\pm j3.27$, $-0.25\pm j6.27$, $-11.4\pm j6.45$, $-0.411\pm j17.2$, $-7.14\pm j19.4$, $-0.349\pm j33.9$, $-4.26\pm j34.1$, $-0.439\pm j55.7$, $+0.11\pm j83.8$, $-0.02\pm j117.04$. This pole shifting due to spillover is quite substantial and results in one of the residual modes being unstable; thus, some type of spillover compensation would be required for successful operation of the active controller.

The implementation of the active controller, Eqs. (8) and (9), for this model with the given actuator and sensor configuration was carried out on a hybrid computer. The controller was implemented digitally with its own separate clock. Experiments were performed to determine the effect of small control delays and nonlinearities as well as sampling error. Sampling error produced aliasing and the observation spillover effects were greatly enhanced; the sample-hold circuits cause spectral spreading which tends to increase the control spillover (for details see Ref. 5).

These experiments show that active control of large spacecraft is possible with on-board computers. As long as the problem of spillover is taken into account, the systems can be easily designed and digitally implemented to accurately point the spacecraft and to suppress structural vibrations. Although no structural damping is present in Eq. (1), the methodology described here can be directly applied to cases where a small amount of damping is present.[4] The situation for control of a large spinning spacecraft, where large amounts of gyroscopic damping are present, has been considered elsewhere.[8-10]

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AIAA 81-4246

Sensitivity of Modal-Space Control to Nonideal Conditions

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Introduction

THE theoretical concept of active modal-space control, with application proposed for flexible space structures, has been developed recently by Meirovitch and Oz.[1-4] The present authors have investigated modal-space control from the perspective of a similar technology, modal vibration testing of structures, and have conducted numerical studies in an attempt to assess the practical applicability of the method.[5,6]

Much of the previous study of modal-space control has been based on certain ideal conditions: knowledge of the exact modal parameters of the structure to be controlled; availability of an essentially unlimited number of control actuators; and the capability of measuring without error the response contribution of an individual mode of vibration. Since these ideal conditions generally do not exist in reality, the practical value of modal-space control depends on its effectiveness under more realistic, nonideal conditions.

Physical and Modal Equations of Motion

References 1-4 employ structure discretization by means of the global assumed modes method. However, we assume here discretization by the more direct finite-element method, for which the basic time-dependent coordinates are displacements of selected grid points on the structure.

The linear deformational and rigid body dynamics of an undamped structure with no gyroscopic members can be described by the physical equations of motion

$$m\ddot{q} + kq = c \tag{1}$$

Received Jan. 23, 1981.

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Square matrices m and k are mass and stiffness matrices, respectively; $q(t)$ is the vector of all grid-point displacements (translations and rotations); and $c(t)$ is the vector of all grid-point control actions (forces and moments). Disturbance excitation is omitted because it has no role in this analysis.

The simplify the analysis, we neglect internal passive structural damping. For the slight structural damping which might be expected of large space structures, this should lead in most cases to conservative predictions of overall (i.e., active plus passive) damping.

We denote the dimension of the vectors in Eq. (1) as N and specify simply that N be as large as necessary to accurately model the dynamics of the structure. Since m and k are assumed to be developed by finite-element analysis, the maximum computationally practical dimension is on the order of a few thousand.

Let us assume that n normal modes, accounting for both rigid body and deformational motion, result from eigenanalysis of Eq. (1) with null right-hand side, that these calculated modes accurately represent the modes of the real structure, and that the number n of modes is adequate to describe all disturbed motion of practical importance. (The possible consequences of inaccurate modal information are discussed in a subsequent section.) We denote the natural frequency of the jth mode as ω_j and the $N\times n$ modal matrix as Φ, the jth column of which ϕ_j is the physical mode shape of the jth mode. The orthogonality conditions are $\Phi^T m\Phi = M = \text{diag}\ (M_1, M_2, \ldots, M_n)$ and $\Phi^T k\Phi = K = \text{diag}\ (K_1, K_2, \ldots, K_n)$, where $K_j = M_j\omega_j^2$. The standard modal transformation of variables is $q = \Phi\xi$, where $\xi(t)$ is the vector of n modal coordinates. With this transformation, premultiplication by Φ^T, and the orthogonality conditions, Eq. (1) becomes

$$M\ddot{\xi} + K\xi = \Phi^T c \tag{2}$$

Modal-Space Control with a Limited Number of Actuators

There can be only a finite number n_a of control actuators, which for a real structure is probably smaller than n. We consider the actuators to be applied at points and in directions on the structure corresponding to a specific subset q^a of degrees of freedom. Accordingly, the $n_a \times 1$ actuator submatrix of c is denoted as c^a, all other elements of c being zero. Hence, in Eq. (2), $\Phi^T c = \Phi^{aT} c^a$, where the $n_a \times n$ matrix Φ^a consists of the appropriate rows of Φ.

Control vector c^a depends on the measured motion, so it can be considered in general a function of all ξ_j's and/or their derivatives. To simplify the analysis, we consider only velocity feedback, so that $c^a = c^a(\dot{\xi})$.

The essence of modal-space control is selection of c^a so as to decouple as much as possible of the right-hand side of Eq. (2). To do this, we first designate n_c specific modes as the modes to be controlled, the remaining n_r modes being the uncontrolled or residual modes. In practice, we would expect $n_a \le n_c < n$. The subset of Eq. (2) describing the controlled modes is

$$M^c\ddot{\xi}^c + K^c\xi^c = \Phi^{acT} c^a(\dot{\xi}) \tag{3}$$

where superscript c denotes appropriate partitions of the matrices in Eq. (2). In particular, the $n_a \times n_c$ matrix Φ^{ac} consists of the columns of Φ^a associated with the controlled modes.

Now we seek for a controlled mode, say mode s, a time-independent shape, c_s^{ac}, of the control vector which will isolate that mode from all other controlled modes. ("Force apportioning" is a term which has been used to describe this process in multiple-shaker modal vibration testing.) Next, we form the total control vector as a linear sum of vectors for all individual controlled modes,